Drift Errors in Passive Remote Wireless SAW Sensing with Multiple DPM

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***Abstract*— We address a probabilistic analysis of drift errors in passive remote wireless surface acoustic wave sensing with multiple differential phase measurement. The rigorous probability density of the differential phase difference is derived and its particular functions, all having no closed forms, are given for different signal-to-noise ratios in the received radio frequency pulses. Employing the von Mises/Tikhonov distribution, an efficient approximation is found via the modified Bessel functions of the first kind and zeroth order. Engineering features and small errors of the approximation are demonstrated. An analysis is given of the phase difference drift rate and error probability for the drift rate to exceed a threshold.**

I. INTRODUCTION

Wireless passive remote surface acoustic wave (SAW) sensing is now used to measure different physical quantities such as temperature, pressure and torque, vehicle velocity at high speed, etc. A review of different methods employing this kind of measurement systems is given in [1]. The quantity is often measured via the phase of the SAW sensor response at the coherent receiver. The phase is estimated using the maximum likelihood function approach and, in order to compensate the environment-caused and propagation errors, differential phase measurement (DPM) is commonly used.

In wireless SAW sensing with DPM, multiple interrogation by pulse-bursts is employed to increase the signal-to-noise ratio (SNR) at the coherent receiver [1]. Because a measured quantity is not always constant during the pulse-burst length, drift errors occur. On the other hand, multiple DPM can be used to measure the drift rate of the quantity or acceleration when the system is intended to measure velocity.

A probabilistic analysis of errors in wireless SAW sensing with DPM has recently been provided in [2], [3]. Although, it gives designers a tool to optimize the interrogating signal power and reach minimum measurement errors thereby, the problem of minimizing effects of drift errors still remains unsolved. In this paper, we provide a probabilistic analysis of the drift errors in the passive remote wireless SAW sensor system with multiple DPM.

II. PRINCIPLES OF WIRELESS SAW SENSING WITH MULTIPLE DPM AND PROBLEM FORMULATION

The operation principle of passive wireless SAW sensing with multiple DPM is illustrated in Fig. 1, in which the carrier is omitted in the radio frequency (RF) pulses. An interrogator transmits to the SAW sensor a RF pulse-burst of K pulses, period T, peak-power $2S$, carrier frequency f_0, and initial phase φ_0. While propagating, a signal acquires an additional phase shift $\phi(t)$ cased by RF wave propagation, Doppler effect, and frequency shift between the carrier of the transmitted signal and resonance frequency of the SAW sensor interdigital transducer (IDT). The latter converts the electric signal to SAW, and about half of its energy distributes to the reflectors R_1 and R_2. The SAW propagates on the piezoelectric crystal surface with a velocity v through double distances ($2L_1$ and $2L_2$), attenuates (6 dB per μs delay time [4]), reflects partly from the reflectors, and returns back to the IDT. Inherently, the SAW undergoes phase delays on the piezoelectric crystal surface causing time shifts $\tau_{2k-1} = \frac{2L_1}{v}$ and $\tau_{2k} = \frac{2L_2}{v}$, $k \in [1, K]$, and information bearing phase shifts, respectively,

$$\psi_{2k-1} = 4\pi f_0 \frac{L_1}{v} \quad \text{and} \quad \psi_{2k} = 4\pi f_0 \frac{L_2}{v}\,. \tag{1}$$

The returned SAW is reconverted by the IDT to the electric signal and retransmitted to the interrogator as a burst of K pairs of RF pulses. Accordingly, the phases ϑ_{2k-1} and ϑ_{2k}, $k \in [1, K]$, of the first and second pulses in the pairs become, respectively,

$$\vartheta_{2k-1} = \varphi_0 + \phi_{2k-1} - \psi_{2k-1}\,, \tag{2}$$

$$\vartheta_{2k} = \varphi_0 + \phi_{2k} - \psi_{2k}\,. \tag{3}$$

To compensate effect of the nuisance factors, systems with DPM employ the interdistance time $\Delta\tau_{(2k)(2k-1)} = \tau_{2k} - \tau_{2k-1} = \frac{2(L_2-L_1)}{v}$ and the phase difference[1]

$$\begin{aligned}\bar{\Theta}_k &= \vartheta_{2k} - \vartheta_{2k-1} &\text{(4a)}\\ &= -\psi_{2k} + \psi_{2k-1} &\text{(4b)}\\ &= -2\pi f_0 \frac{L_2 - L_1}{v}\,. &\text{(4c)}\end{aligned}$$

The time drift in $\bar{\Theta}_k$ is evaluated via the DPD

$$\begin{aligned}\bar{\Psi}_k &= \bar{\Theta}_k - \bar{\Theta}_{k-1} &\text{(5a)}\\ &= -\psi_{2k} + \psi_{2k-1} + \psi_{2k-2} - \psi_{2k-3}\,. &\text{(5b)}\end{aligned}$$

[1]For the sake of simplicity, we assume equal phase delays in the channel letting $\phi_{2k-1} = \phi_{2k}$. If $\phi_{2k-1} \neq \phi_{2k}$, the value $\phi_{2k} - \phi_{2k-1}$ can be measured and accounted for as a regular error.

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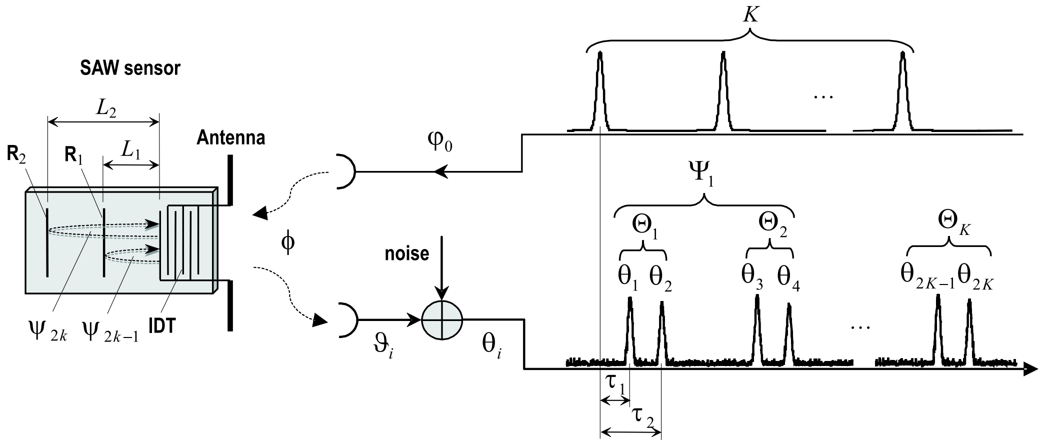


Fig. 1. Operation principle of passive remote wireless SAW sensing with multiple DPM.

At the coherent receiver [5], the RF pulse-burst is contaminated by narrowband Gaussian noise having the variance σ^2 and known power spectral density. The phase $\vartheta_i \in [\vartheta_{2k-1}, \vartheta_{2k}]$, $i \in [1, 2K]$, (2) and (3), is thus represented at the receiver detector with the noisy phase θ_i, which has Bennett's pdf [6]. For signals affected by fading and fluctuations, the conditional form of this pdf is [3]

$$p(\theta_i|\gamma_i, \vartheta_i) = \frac{e^{-\gamma_i}}{2\pi} + \sqrt{\frac{\gamma_i}{\pi}} e^{-\gamma_i \sin^2 \tilde{\theta}_i} \times \Phi\left(\sqrt{2\gamma_i}\cos\tilde{\theta}_i\right)\cos\tilde{\theta}_i\,, \quad (6)$$

where $\tilde{\theta}_i = \theta_i - \vartheta_i$ is the modulo 2π phase[2], $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}\mathrm{d}t$ is the probability integral,

$$\gamma_i = \frac{S\alpha_i}{\sigma^2} \quad (7)$$

is the SNR in the received signal i-pulse, and α_i is the peak-power correction coefficient caused by fading and signal attenuation. It has been shown in [7], [8] that (6) is fundamental for the interrogating RF pulses of arbitrary waveforms and modulation laws. Employing the maximum likelihood function approach, the coherent receiver produces an estimate $\hat{\theta}_i$ of θ_i [2]. Assuming in this paper an ideal receiver, we let $\hat{\theta}_i = \theta_i$.

In line with θ_i, the information bearing phase difference $\bar{\Theta}_k$ (4b) is represented at the receiver detector with its noisy version Θ_k. If the SNRs in the received pulses are equal, $\gamma = \gamma_{2k-1} = \gamma_{2k}$, then Θ_k has conditional Tsvetnov's pdf [2], [9]

$$p(\Theta_k|\gamma, \bar{\Theta}_k) = \frac{e^{-\gamma}}{2\pi} + \frac{\gamma e^{-\gamma}}{2\pi} \times \int_0^{\pi/2} \left(\cos z + \cos\tilde{\Theta}_k\right) e^{\gamma\cos\tilde{\Theta}_k \cos z}\mathrm{d}z\,, \quad (8)$$

where $\tilde{\Theta}_k = \Theta_k - \bar{\Theta}_k$. For different SNRs, $\gamma_{2k-1} \neq \gamma_{2k}$, the relevant pdf was found by Tsvetnov in [10]. In [11], Pawula showed an equivalent form of this pdf, namely

$$p(\Theta_k|\gamma_{2k-1}, \gamma_{2k}, \bar{\Theta}_k) = \frac{e^{-\bar{\gamma}}}{2\pi}\Big[\cosh\breve{\gamma} + \frac{1}{2}\int_0^{\pi} (\bar{\gamma}\sin y + \lambda)\cosh(\breve{\gamma}\cos y) \times e^{\lambda \sin y}\mathrm{d}y\Big]\,, \quad (9)$$

where $\bar{\gamma} = (\gamma_{2k-1} + \gamma_{2k})/2$, $\breve{\gamma} = (\gamma_{2k} - \gamma_{2k-1})/2$, $\xi = \arctan(\breve{\gamma}/\lambda)$, and $\lambda = \sqrt{\gamma_{2k-1}\gamma_{2k}}\cos\tilde{\Theta}_k$. By equal SNRs, (9) becomes (8).

It has to be remarked now that (9) can be derived via (6) using the Rice substitution of variables [12], $\theta_{2k-1} = \theta$ and $\theta_{2k} = \theta + \Theta_k$, by

$$p(\Theta_k|\gamma_{2k-1}, \gamma_{2k}, \bar{\Theta}_k) = \int_{-\pi}^{\pi} p(\theta + \Theta_k|\gamma_{2k}, \bar{\Theta}_k)p(\theta|\gamma_{2k-1})\mathrm{d}\theta\,. \quad (10)$$

On the other hand, (10) imposes no restrictions upon the θ mod 2π and can thus be applied to any angular measure.

The problem now formulates as follows. At the coherent receiver, the DPD $\bar{\Psi}_k$ (5b) becomes noisy, Ψ_k, in the presence of Gaussian noise. We would like to derive the pdf of Ψ_k for different SNRs in the pulses and find engineering approximations. We then would like to investigate the mean and variance

[2]Through the paper, we consider the modulo 2π phases, phase differences, and DPDs existing from $-\pi$ to π as associated with wireless systems.

of the drift rate and the error probability for the drift rate to exceed a threshold with applications to wireless SAW sensing.

III. Probability Density of the DPD

To derive the pdf of the DPD Ψ, which actual value is given by (5b), one can start with (10) viewing it as the pdf of the phase difference $\Psi_k = \Theta_k - \Theta_{k-1}$ and substituting the integrand with $p(\Theta + \Psi_k|\gamma_{2k-1}, \gamma_{2k}, \bar{\Psi}_k)$ and $p(\Theta|\gamma_{2k-3}, \gamma_{2k-2})$. The derivation of two forms of this pdf is given in [13]. It has also been shown in [13] that there is no alternative to the rigorous pdf forms when the SNRs are different. Notwithstanding this fact, the rigorous functions do not demonstrate engineering features. Below, we shall show that this disadvantage can be circumvented with a simple and quite accurate approximating function.

It has been shown in [14] that the von Mises/Tikhonov distribution fits the phase difference with the maximum approximation error lesser than 0.58% if Tikhonov's pdf[3]

$$p_T(\phi|\gamma, \phi_0) = \frac{e^{\alpha \cos(\phi - \phi_0)}}{2\pi I_0(\alpha)}\,, \tag{11}$$

where

$$\alpha(\gamma) = \gamma(1 + a e^{-b\gamma})\,, \tag{12}$$

$a = 0.525$, and $b = 1.1503$, is used instead of Tsvetnov's density (9). The Tikhonov-based approximation $\bar{p}_\Psi$ for p_Ψ given by either [13, eq. (I.3)] or [13, eq.(I.5)] appears if to set $\phi = \Psi_k$, $\phi_0 = \bar{\Psi}_k$, $\tilde{\Psi}_k = \Psi_k - \bar{\Psi}_k$, and apply (11) to (10). We thus have

$$\begin{aligned} \bar{p}_\Psi &\triangleq \bar{p}(\Psi_k|\gamma_{2k}, \gamma_{2k-1}, \gamma_{2k-2}, \gamma_{2k-3}, \tilde{\Psi}_k) \\ &= \int_{-\pi}^{\pi} p_T(\Theta + \Psi_k|\gamma_{2k-2}, \gamma_{2k-3}, \tilde{\Psi}_k) \\ &\quad \times p_T(\Theta|\gamma_{2k}, \gamma_{2k-1}) \mathrm{d}\Theta \\ &= \frac{1}{2\pi} \frac{I_0(r)}{I_0(\alpha_1) I_0(\alpha_2)}\,, \end{aligned} \tag{13}$$

where $\alpha_1 \equiv \alpha(\bar{\gamma}_1)$, $\alpha_2 \equiv \alpha(\bar{\gamma}_2)$,

$$r(\Psi, \alpha_1, \alpha_2) = \sqrt{\alpha_1^2 + 2\alpha_1\alpha_2 \cos\tilde{\Psi} + \alpha_2^2}\,,$$

$$\bar{\gamma}_1 = \frac{2\gamma_{2k}\gamma_{2k-1}}{\gamma_{2k} + \gamma_{2k-1}}\,,$$

$$\bar{\gamma}_2 = \frac{2\gamma_{2k-2}\gamma_{2k-3}}{\gamma_{2k-2} + \gamma_{2k-3}}\,,$$

and both $\alpha(\bar{\gamma}_1)$ and $\alpha(\bar{\gamma}_2)$ are specified by (12). In applications of (13) to wireless SAW sensing with multiple DPM, the following cases are of interest:

- Large SNR in one of the pulses, e.g., $1 \ll \bar{\gamma}_2$ and $\bar{\gamma}_1 \ll \bar{\gamma}_2$, although no one pulse is lost. With large $\bar{\gamma}_2$, one may set $\alpha_2 \cong \bar{\gamma}_2$, use an approximation of the modified Bessel functions $I_n(x)|_{x \gg 1} \cong \frac{e^x}{\sqrt{2\pi x}}$, and first go to $I_0(\alpha_2) \cong I_0(\bar{\gamma}_2) \cong \frac{e^{\bar{\gamma}_2}}{\sqrt{2\pi\bar{\gamma}_2}}$ and
$$I_0(r) \cong \frac{e^{\sqrt{\alpha_1^2 + 2\alpha_1\bar{\gamma}_2\cos\tilde{\Psi}_k + \bar{\gamma}_2^2}}}{\sqrt{2\pi\sqrt{\alpha_1^2 + 2\alpha_1\bar{\gamma}_2\cos\tilde{\Psi}_k + \bar{\gamma}_2^2}}}\,,$$
where $r = \sqrt{\alpha_1^2 + 2\alpha_1\bar{\gamma}_2\cos\tilde{\Psi}_k + \bar{\gamma}_2^2}$. The ratio $I_0(r)/I_0(\alpha_2)$, after the transformations and neglecting products of small values, becomes $e^{\alpha_1 \cos\tilde{\Psi}_k}$ causing (13) to degenerate to (11).
- One of the pulses is lost (pure noise), e.g., $\bar{\gamma}_2 = 0$ and $0 < \bar{\gamma}_1$. Substituting $\alpha_2 = 0$, $I_0(\alpha_2) = 1$, and $I_0(r) = I_0(\alpha_1)$ makes (13) uniform, disregarding the value of γ_1; that is,
$$\bar{p}(\Psi_k) = \frac{1}{2\pi}\,. \tag{14}$$
- Large and equal SNRs in the pulses, e.g., $1 \ll \bar{\gamma} = \bar{\gamma}_1 = \bar{\gamma}_2$. In this case, (13) degenerates to the Gaussian law
$$\bar{p}(\Psi_k|\bar{\gamma}, \bar{\Psi}_k) = \sqrt{\frac{\bar{\gamma}}{4\pi}} e^{-\frac{\bar{\gamma}}{4}\tilde{\Psi}_k^2}\,. \tag{15}$$

Figure 2 shows how well (13) fits either [13, eq.(I.3)] or [13, eq.(I.5)] in terms of the fractional error $\varepsilon(\Psi), \% = 100\frac{p(\Psi_k) - \bar{p}(\Psi_k)}{p(\Psi_k)}$ with $\bar{\Psi}_k = 0$. We first set $\gamma = \gamma_1 = \gamma_2 = 1$ and very the SNR in the second pulses, γ_{2k}, from 1 to ∞ (Fig. 2a). As can be seen, a maximum error lesser 0.45% occurs when $\gamma_{2k} \to \infty$. It reduces to 0.41% with $\gamma = 1$, although with $\gamma = 2$ and $\gamma = 3$ the error takes values of 0.19% and 0.15%, respectively. Allowing $\gamma_{2k} = 50$ and changing $\gamma = \gamma_1 = \gamma_2$ from 0.1 to 50, one infers that the error reduces substantially by increasing γ and becomes insignificant when γ reaches 50. The same tendency is kept by reducing γ to zero. It can also be observed that the maximum possible error of about 0.63% corresponds to $\gamma = 0.5$. Note that the latter case is not associated with normal functioning of wireless SAW systems and one can rely on much lower approximation errors.

Based upon the above derived approximate distributions, we shall now find and investigate engineering relations for two most important measures associated with the DPD: the phase difference drift rate and error probability for this rate to exceed an allowed value.

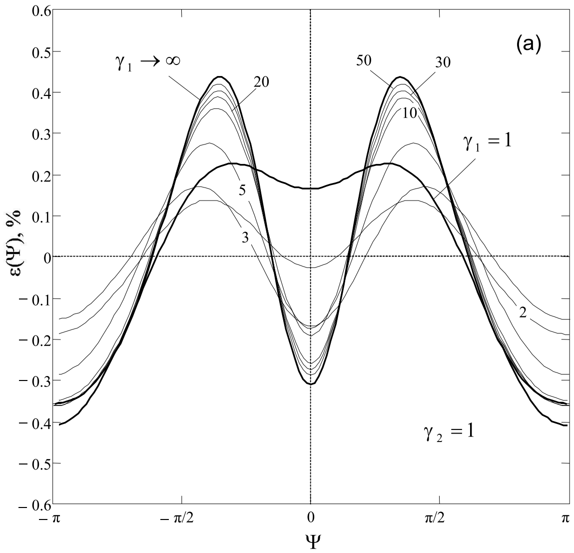
IV. Drift Rate

A measure D of the drift rate of the phase difference Θ_k in the received RF pulse-burst has three critical applications:

- It represents the drift rate error when the burst is used to increase the SNR in the received signal [1].
- When the SAW sensor is intended to measure a physical quantity, then D characterizes speed of change of this quantity.
- If the SAW system measures velocity of a moving object, then D gives a measure of acceleration.

In applications, of interest may be the mean value $\langle D \rangle$ and variance σ_D^2 of the drift rate.

[3]Tikhonov derived his distribution in [15] as an approximation for the phase of the first order phase locked loop.



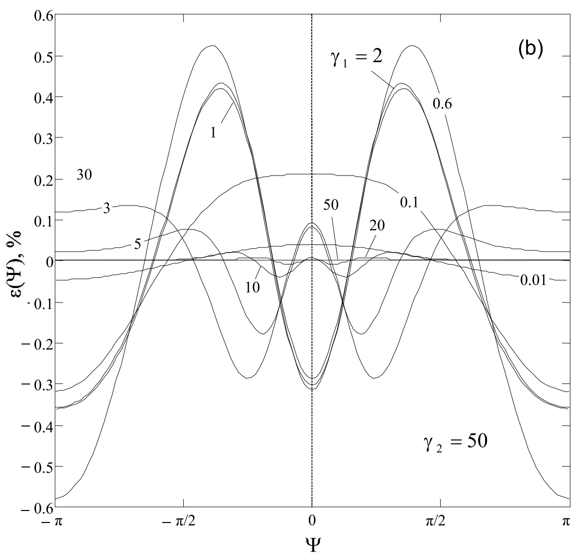


Fig. 2. Approximation error $\varepsilon(\Psi),\% = 100\frac{p(\Psi_k)-\bar{p}(\Psi_k)}{p(\Psi_k)}$ for $\bar{\Psi}_k = 0$ and different γ_2: (a) $\gamma_2 = 1$ and $1 \leqslant \gamma_1 \leqslant \infty$ and (b) $\gamma_1 = 50$ and $0.1 \leqslant \gamma_2 \leqslant 50$.

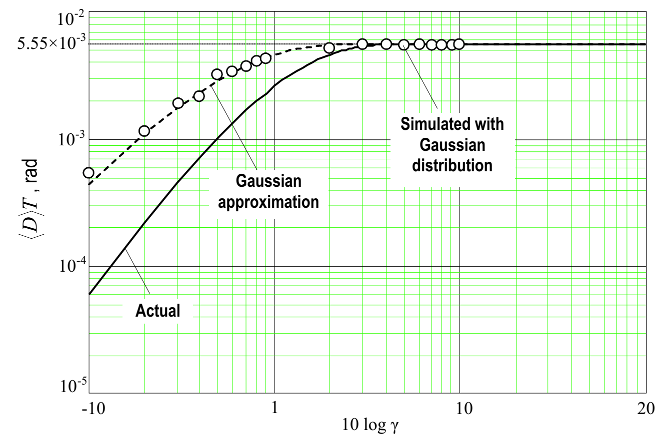


Fig. 3. The mean drift rate $\langle D\rangle$ for $\bar{\Psi} = 5.55 \times 10^{-3}$ rad: actual, by (16); Gaussian approximation, by (15); and simulation with Gaussian distribution, using (17).

A. Mean Drift Rate

The mean drift rate $\langle D\rangle$ can be evaluated via (13) and the mean value $\langle \Psi_k\rangle$ given in [14, eq.(13)] as in the following,

$$\begin{aligned}\langle D\rangle &= \frac{\langle\Psi_k\rangle}{T} \\ &= \frac{1}{T}\int_{-\pi}^{\pi} \Psi_k \bar{p}(\Psi_k|\gamma_{2k},\gamma_{2k-1},\gamma_{2k-2},\gamma_{2k-3},\tilde{\Psi}_k)\mathrm{d}\Psi_k \\ &= \frac{2}{T}\sum_{n=1}^{N}\frac{(-1)^{n+1}}{n}\frac{I_n(\alpha_1)}{I_0(\alpha_1)}\frac{I_n(\alpha_2)}{I_0(\alpha_2)}\sin n\bar{\Psi}_k\,, \qquad (16)\end{aligned}$$

where $I_i(x)$ is the modified Bessel functions of the first kind and ith order and a reasonable series length is limited with $N \geqslant 2\max\bar{\gamma}_{1,2}$ [2]. The estimate of $\langle D\rangle$ can be found by averaging the estimates $\hat{\Psi}_k$ obtained at the coherent receiver,

$$\langle\hat{D}\rangle = \frac{1}{TK}\sum_{k=1}^{K}\hat{\Psi}_k\,. \qquad (17)$$

1) Example 1: Burst length vs. the drift rate: Consider a passive SAW sensor of temperature [16] operating at the frequency $f_0 = 2.45$GHz with the temperature sensitivities of the delay time difference $S_t = 0.017$ ns/K and phase difference $S_p = 2\pi f_0 S_t = 0.262$ rad/K. Suppose that the temperature rate at the sensor substrate is 1 K per 10 sec; that is $R_t = 10^{-10}$ K/ns. The phase difference mean drift rate is thus $\langle D\rangle = S_p R_t = 2.62 \times 10^{-11}$ rad/ns.

The sensor is interrogated with the pulse-burst of K pulses and period T. During the burst length $L = KT$, temperature is changed at $\Delta T = R_t L$ K and the phase difference at $\Delta\Theta = \langle D\rangle L$ rad. For the allowed error of $\epsilon = 0.1^\circ$ in the temperature range of $T_r = 300^\circ$, the mean phase error is $\epsilon_p = \pi\epsilon/T_r = 1.047 \times 10^{-3}$ rad. By $\Delta\Theta = \epsilon_p$, the pulse-burst length is thus limited with

$$L \leqslant \frac{\pi\epsilon}{\langle D\rangle T_r} \times 10^{-9} = 0.04\,\text{sec}\,.$$

2) Example 2: Effect of the SNR on the mean drift rate: Figure 3 shows the effect of the SNR, by equal values γ in each of the pulses, on $\langle D\rangle$ for $\bar{\Psi} = 5.55 \times 10^{-3}$ rad. Actual values are calculated by (16) for $\gamma \leqslant 20$ and by (15) when $\gamma > 20$. Supposing that Ψ is distributed with (15) over all values of γ, we arrive at an approximation (dashed). For the latter case, the process was simulated and $\langle D\rangle$ calculated numerically (circles), by (17). As can be seen, the approximation errors practically vanish when γ exceeds 4 dB. Otherwise, bias occurs in the estimate. Table I gives the relevant values for $\bar{\Psi}$ ranging from 0.1π to 0.9π. A simple measure of accuracy used here is when the exact and approximate values become visually indistinguishable.

TABLE I

ALLOWED γ FOR ACCURATE ESTIMATION OF $\langle D\rangle$ WITH DIFFERENT $\bar{\Psi}$

γ	$\lvert\bar{\Psi}\rvert$, rad					
	0.1π	0.3π	0.5π	0.7π	0.8π	0.9π
dB	>4	>5	>7	>11	>13	>21

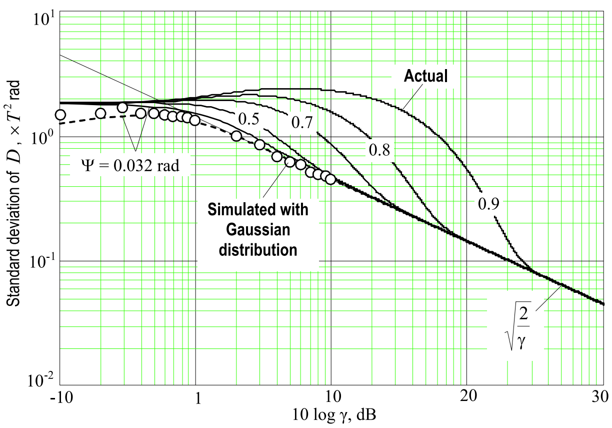


Fig. 4. The standard deviation $\sqrt{\sigma_D^2}$ for different $\bar{\Psi}$: actual (bold), by (18); Gaussian approximation (dashed) for $\bar{\Psi} = 0.032$ rad; and simulation with Gaussian distribution, using (19).

B. Drift Rate Variance

The variance of the drift rate is provided, using (16), via the mean square value found in [14, eq.(14)] to be

$$\begin{aligned}\sigma_D^2 &= \frac{\langle\Psi_k^2\rangle}{T^2} - \langle D\rangle^2 \\ &= \frac{\pi^2}{3T^2} + \frac{4}{T^2}\sum_{n=1}^{N}\frac{(-1)^n}{n^2}\frac{I_n(\alpha_1)}{I_0(\alpha_1)}\frac{I_n(\alpha_2)}{I_0(\alpha_2)}\cos n\bar{\Psi}_k \\ &\quad -\langle D\rangle^2\,, \end{aligned} \tag{18}$$

and its estimate is obtained by averaging as

$$\hat{\sigma}_D^2 = \frac{1}{TK}\sum_{k=1}^{K}(\hat{\Psi}_k - \langle D\rangle)^2\,. \tag{19}$$

Figure 4 sketches the standard deviation $\sqrt{\sigma_D^2}$ evaluated by (18) and (19) for different values of $\bar{\Psi}$. Along, we show the estimates (dashed) and simulated values calculated by (19) for $\bar{\Psi} = 0.032$ rad assuming Gaussian approximation (15). For the comparison with the mean drift rate (Table I), Table II gives the minimum values of γ, for several $\bar{\Psi}$, allowing for accurate evaluation of σ_D^2.

TABLE II

ALLOWED γ FOR ACCURATE EVALUATION OF σ_D^2 WITH DIFFERENT $\bar{\Psi}$

γ	$\lvert\bar{\Psi}\rvert$, rad					
	0.1π	0.3π	0.5π	0.7π	0.8π	0.9π
dB	>8	>9	>10	>13	>18	>23

An important inference follows instantly. For the sake of minimum errors, the SNR in the pulses must be obtained larger than 23 dB for $\bar{\Psi}$ ranging from -0.9π to 0.9π. We notice that similar values were found in [2] and [3] for the phase difference Θ.

1) Cramér-Rao lower bound: An alternative measure of $\hat{\sigma}_D^2$ is the Cramér-Rao lower bound (CRLB) found in [3] for wireless SAW sensing with multiple DPM regarding the phase difference. An analysis shows that for multiple DPM, the CRLB cannot be found in simple functions and the best candidate for the estimate of σ_D^2 still remains (19).

V. ERROR PROBABILITY FOR THE DRIFT RATE TO EXCEED A THRESHOLD

In applications, it might be required for the phase difference drift rate to range below some allowed value. The relevant error probability P_E can be characterized by the probability for the DPD to exceed a threshold ζ. Because the pdf of the modulo 2π angular measure is 2π-periodical, the P_E is commonly ascertained by setting $\bar{\Psi}_k = 0$. Using (13), we thus have

$$\begin{aligned}P_E &\triangleq P_E(\zeta|\bar{\gamma}_1,\bar{\gamma}_2) \\ &= 2\int_{\zeta}^{\pi}\bar{p}(\Psi_k|\bar{\gamma}_1,\bar{\gamma}_2)\mathrm{d}\Psi_k \qquad (20\text{a}) \\ &= \frac{1}{\pi I_0(\alpha_1)I_0(\alpha_2)} \\ &\quad\times\int_{\zeta}^{\pi} I_0\left(\sqrt{\alpha_1^2 + 2\alpha_1\alpha_2\cos z + \alpha_2^2}\right)\mathrm{d}z\,. \qquad (20\text{b})\end{aligned}$$

Expanding the integrand in (20b) to the Fourier series [17],

$$I_0\left(\sqrt{\alpha_1^2 + 2\alpha_1\alpha_2\cos z + \alpha_2^2}\right) = \sum_{n=0}^{\infty}\epsilon_n I_n(\alpha_1)I_n(\alpha_2)\cos nz\,,$$

where $\epsilon_0 = 1$ and $\epsilon_{n>0} = 2$, brings (20b) to several useful estimates

$$\begin{aligned}P_E &= \sum_{n=0}^{\infty}\frac{\epsilon_n}{\pi n}\frac{I_n(\alpha_1)}{I_0(\alpha_1)}\frac{I_n(\alpha_2)}{I_0(\alpha_2)}(\sin n\pi - \sin n\zeta) \qquad (21\text{a}) \\ &\cong 1 - \frac{\zeta}{\pi}\left[1 + 2\sum_{n=1}^{N}\frac{I_n(\alpha_1)}{I_0(\alpha_1)}\frac{I_n(\alpha_2)}{I_0(\alpha_2)}\frac{\sin n\zeta}{n\zeta}\right] \qquad (21\text{b}) \\ &\cong 1 - \frac{\zeta}{\pi}\left[1 + 2\sum_{n=1}^{N}\frac{I_n(\alpha_1)}{I_0(\alpha_1)}\frac{I_n(\alpha_2)}{I_0(\alpha_2)}\right],\ \zeta \ll \pi\,, \qquad (21\text{c}) \\ &\cong 1 - \frac{\zeta}{\pi}(1 + 2N)\,,\quad \zeta \ll \pi,\quad 1 \ll \bar{\gamma}_{1,2}\,, \qquad (21\text{d}) \\ &\leqslant 1 - \frac{\zeta}{\pi}(1 + 4\max\bar{\gamma}_{1,2})\,,\ \zeta \ll \pi,\ 1 \ll \bar{\gamma}_{1,2}\,. \qquad (21\text{e})\end{aligned}$$

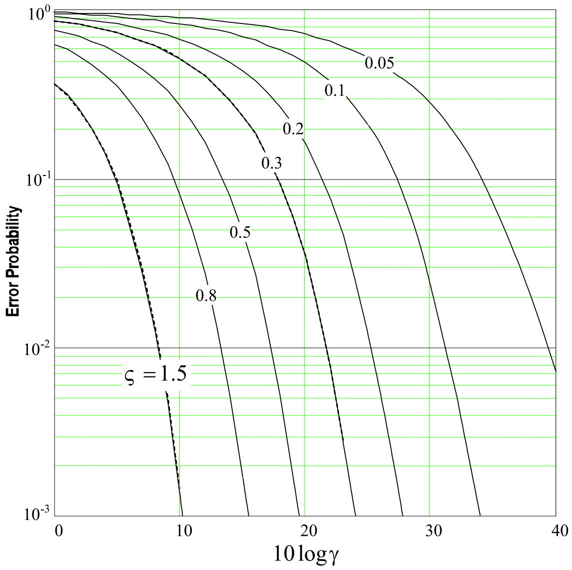


Fig. 5. Error probability for the drift rate to exceed a limit ζ with equal SNRs in the pulses: rigorous (dashed), by (**??**), and approximate (bold), by (21b).

Figure 4 illustrates the error probability calculated rigorously (dashed), by [13, eq.(I.5)] and (20a), and approximately (bold), by (21b) for equal SNRs in the pulses. One infers that the approximation error is negligibly small in the whole range of angular measures.

VI. Concluding Remarks

This study addresses a statistical analysis of drift errors in passive remote wireless SAW sensing with multiple DPM. We derived the pdf of the DPD in the presence of Gaussian noise and showed that the function involves double integration having no closed form. Because it must be quite difficult to use this pdf in engineering tasks, we found a simple and reasonably accurate approximation via the well-known modified Bessel functions of the first kind. Based upon, we investigated the drift errors that allowed us to make the following generalizations:

- The Tikhonov-based approximation (13) of the pdf of the DPD, [13, eq.(I.3)] or [13, eq.(I.5)], allows for a maximum error of 0.63% when $\gamma = 0.5$. The error reduces substantially when $\gamma \gg 1$.
- Gaussian approximation (15) allows for negligible errors in the estimates of $\langle D\rangle$ and σ_D^2 over $|\Theta| < 0.9\pi$ if the SNR exceeds 23 dB.
- In multiple measurements of temperature in the range of T_r,K with the pulse-burst length L, the drift rate error ϵ,K caused by $\langle D\rangle$ is calculated by (Example 1):

$$\epsilon = \frac{1}{\pi} L\langle D\rangle T_r \,.$$

Finally, we notice that a measure of D can also serve as an indicator of drift errors in systems with supposedly stationary measurements. Design of the relevant estimation algorithm and some other closely related problems are currently under investigation.

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